JUDGING THE LIMITS OF BATTLE MANAGEMENT IN TBMD

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Abstract

Theater ballistic missile defense (TBMD) systems are required to be able to engage not only single ballistic missile targets but also a number of nearly simultaneous targets in a raid. In many potential situations, the size of the raid may be such that more than one defensive system would be necessary to engage all of the targets. It is generally thought that to effectively engage a raid using multiple systems, a sophisticated battle management, engagement coordination, and data communication system is required. This report considers raids of theater ballistic missiles (TBMs), simultaneously engageable by multiple defensive systems and examines the implicit battle management requirements imposed when weapon system capabilities are defined. Based on the results from a simple model, the conclusion is drawn that system coordination processes must be examined in the overall context of actual requirements and individual weapon system capabilities. Even perfect coordination may not significantly increase the expected overall effectiveness of a system of systems.

Analysis

Consider a raid of N TBMs to be engaged by n defensive systems. Assume that each of the n systems is capable of engaging at most M of the TBMs. (In many cases, $M < N$ [or maybe $M << N$], but that restriction is not imposed here.) If no coordination takes place, each defensive system will independently choose M of the targets for engagement. This selection could result in many potential outcomes ranging from M targets engaged by all n systems and (N-M) targets remaining unengaged to all N targets engaged, with some engaged by more than one of the n systems. All of these outcomes are not equally likely or result in the same level of effectiveness. Although this problem can be addressed using a conventional multiple Urn model, it was found to be simpler to create a straightforward stochastic computer model parameterized with the number of targets and defensive systems. Two particular examples demonstrating possible results are discussed next. The first is a case of small numbers of systems; the second extends the analysis into larger values of N, M and n.

Example 1

Assume that there are four TBMs engageable by three defensive systems (which will be assumed to be ships), each of which has the capability of engaging two TBMs. If no coordination occurs and each ship chooses two TBMs at random, five basic outcomes can occur:

1) Two targets are engaged by all three ships, two targets are unengaged
2) One target is engaged by all three ships, one target is engaged by two ships, one target is engaged by one ship, and one target is unengaged
3) One target is engaged by three ships and three targets are engaged by only one ship each
4) Three targets are engaged by two ships and one target is unengaged

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Abstract

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5) Two ships engage two targets and two targets are engaged by only one ship.

In each case, six engagements take place (three ships times two engagements per ship is six engagements). Each case results in a different expected number of kills and each case is not equally likely.

As an example, we will calculate the expected effectiveness (average probability of negation, P_N) for case 2. Assuming an individual engagement $P_k = 0.7$ and all engagements are independent, we obtain the expected effectiveness for case 2:

One target: [Three shots]
Kill Effectiveness = $1 - (1 - P_k)^3 = 0.973$
One target: [Two shots]
Kill Effectiveness = $1 - (1 - P_k)^2 = 0.91$
One target: [One shot]
Kill Effectiveness = $1 - (1 - P_k)^1 = 0.7$
One target: [Zero shots]
Kill Effectiveness = $1 - (1 - P_k)^0 = 0.0$

Summing these four values and dividing by four, the expected number of kills is 2.583/4 = 0.646. The three ships can choose the four targets and result in this general engagement pattern (3-2-1-0) in 72 different ways. The five cases can be obtained in a total of 216 different ways. These results are summarized in Table 1.

The expected average effectiveness averaged over all possible independent, random assignments is 0.725 with a standard deviation of 0.08.

Now, assume a perfect coordination process. To obtain the best possible overall result, the six possible engagements should be distributed over the four targets as uniformly as possible. Case 5 is this distribution and results in the (highest) expected effectiveness of 0.805. Therefore, we see that perfect coordination raises the average outcome from 0.73 to 0.81 for this particular situation, an increase in average effectiveness of about 8%.

Three additional comments should be made. First, with no coordination, the most likely outcome (about 42% of the cases) is the perfect coordination case (case 5); there is about a 58% chance of getting a less than optimum outcome with no coordination. Second, the worst outcome (case 1) is also the least likely (<3% chance) when no coordination takes place.

Table 1. Four Targets, Three Ships

Case	**A**	**B**	**C**	**D**	**Number Engaged**	**Expected Effectiveness, P_N**	**Probability of Occurrence**
1	XX X	XX X			2	0.486	6/216
2	XX X	XX	X		3	0.646	72/216
3	XX X	X	X	X	4	0.768	24/216
4	XX	XX	XX		3	0.682	24/216
5	XX	XX	X	X	4	0.805	90/216
Total						Average = 0.725	

Third, three of the cases (1, 2, and 4) result in either one or two unengaged targets (probability of raid annihilation or zero leakers is zero).

Looking at this situation from another viewpoint, assume that an overall effectiveness of 0.8 is required. Obviously, the three ships engaging two targets each with perfect coordination (0.81) would meet the requirement whereas the ships operating in an uncoordinated manner (0.725) would not. What if the engagement capability of the ships was increased or if the number of ships was increased? These results are summarized in Table 2.

Table 2.

Situation	Perfect Coordination	Un-coordinated
3 Ships, 2 Engagements each	$\mathbf{P_N = .81}$	$P_N = .73$
3 Ships, 3 Engagements each	$P_N = .93$	$\mathbf{P_N = .89}$
4 Ships, 2 Engagements each	$P_N = .91$	$\mathbf{P_N = .82}$

As can be seen, the average effectiveness of 0.8 can be met by several example approaches: three ships engaging two targets each with perfect coordination, three ships engaging three targets each with no coordination, or four ships engaging two targets each with no coordination. An important total system question to be answered is how the total cost ($, time) of developing a (perfect) battle management and coordination system relates to the cost of improving the capabilities of an individual ship system or acquiring (or assigning) additional ships to counter a mass raid.

Example 2

With this background, a more complex situation is analyzed. Assume the raid now totals seventeen (17) TBMs (N = 17). The ships need to be able to engage at least this number of targets. We could have two ships engage nine (or more) targets each, three ships engage six targets each, four ships engage five targets each, five ships engage four targets each, etc. (Note: coordination is not an issue if there is only one ship.) Figure 1 shows the results for the best outcome that could be obtained with perfect coordination and a single engagement $P_k = 0.7$. As would be expected, the overall effectiveness increases with number of ships and the number of engagements each ship can conduct. Figure 2 shows the average negation probability results for uncoordinated cases, while Figure 3 presents the ratio of the average (uncoordinated) to the maximum. (Those cases where the total number of engagements possible is less than 17 are not important here but were included for completeness.)

As is apparent from Figure 3, uncoordinated engagements result in between 77% and 98% of the maximum effectiveness. As was true in the simpler case, there is a potential variability in the outcomes for uncoordinated engagements. Figure 2 also displays the standard deviations of the results about the expected values as error bars. As can be seen, the results will usually be within 1–4% of the expected values. For completeness, Figure 4 shows the minimum possible values of PN for these cases. The minimum values are obtained when all ships engage the same subset of targets; as Figure 4 shows, these results are not

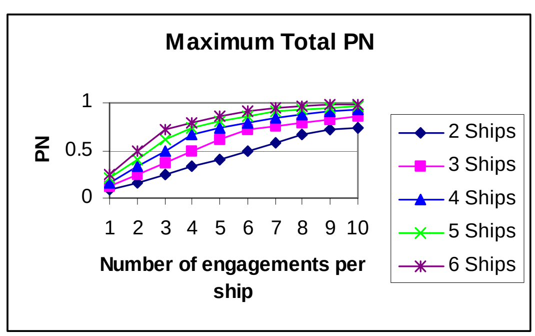

Figure 1

strongly dependent upon the number of ships for the fixed total number of targets. These minimum values could be a result of an uncoordinated engagement but are extremely unlikely. For example, with four ships engaging five targets each with a total of

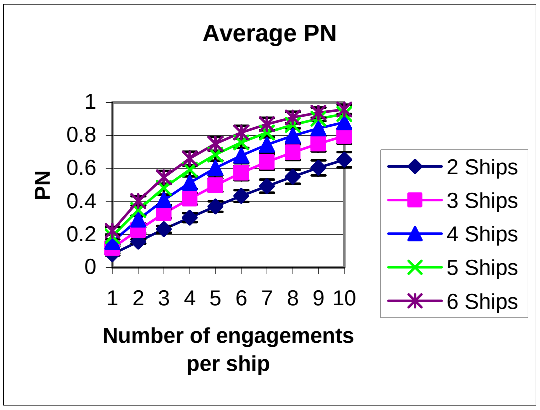

Figure 2.

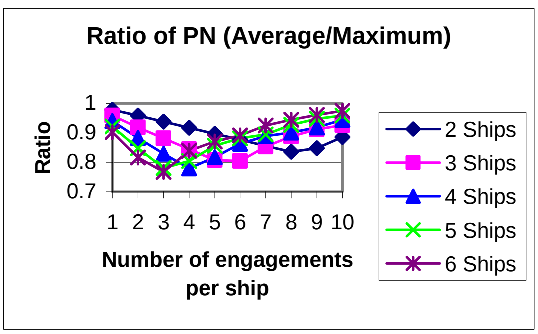

Figure 3.

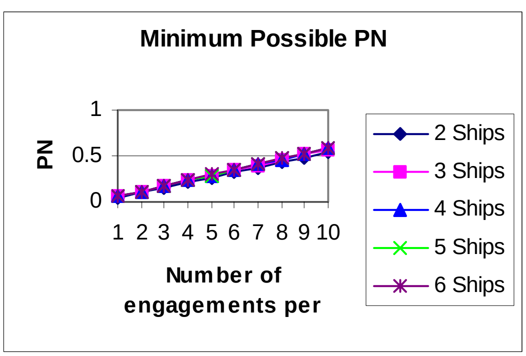

Figure 4.

seventeen targets, the probability of having an uncoordinated engagement resulting in the minimum value is less than one in a billion. As is shown in Figure 5, an average of almost 13 of the 17 targets are engaged, but there is a negligible probability that only a small number of the 17 are engaged. Figure 6 shows how the average engaged fraction of the 17 targets increases with the number of engagements.

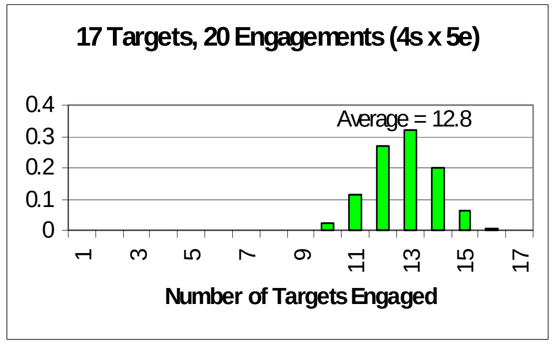

Figure 5.

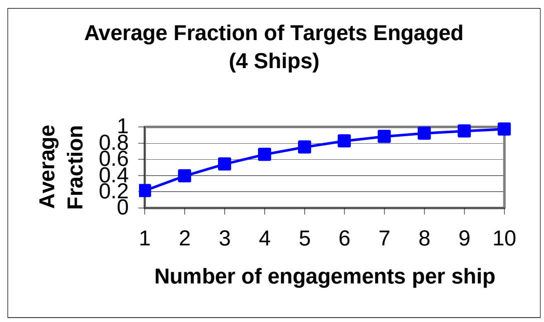

Figure 6.

For this raid size, equivalent average effectiveness can be obtained from several combinations of system capabilities. For example, if a baseline case is four ships engaging five targets each, perfect coordination results in an overall P_N of about 0.74; the expected value of P_N for the uncoordinated case is about 0.60. If the capability of each ship is increased so that each could engage seven targets, the expected value for the uncoordinated case is increased to 0.75, approximately the same value as for the perfectly coordinated baseline. Alternatively, six ships each engaging five targets would also have an expected value for P_N of about 0.75. We therefore see that the lack of perfect coordination can potentially be overcome by additional total system capabilities. In addition, it must be noted that the uncoordinated results could also be improved using imperfect (and simpler) coordination schemes.

Of course, expected effectiveness is not the only measure that can be used to judge total system capabilities; the probability of successfully defeating all of the threats (probability of zero leakers [Pzl]) is another one. Figure 7 shows the values for Pzl for four ships engaging a multiple-target raid with optimum coordination. As is apparent, the probability that there will be no leakers tends to be low for raids of significant size.

The value of Pzl is also a function of the capability of an individual ship to successfully engage a target. Figure 7 shows Pzl for a raid of 17 missiles engaged by four perfectly coordinated ships with several values of individual ship probability of kill, P_k. Notice that Pzl is small for the cases where the ships can barely engage 17 targets (4–5 engagements per ship) unless the individual P_k values are extremely high.

For the same situation, Figure 8 shows the expected averaged P_N over the raid using no coordination. Because of the statistical averaging taking place, the average total P_N is not extremely sensitive to the value of the underlying individual effectiveness. Figure 9 shows the ratio of the average P_N to the maximum P_N (obtained using perfect coordination). As was presented previously, the average effectiveness with no coordination results in between 71% and 96% of the perfectly coordinated effectiveness. It should also be noted that the relative effectivenss generally decreases with increasing individual P_k.

Looking at a subset of the cases, Figure 10 presents how a total system effectiveness requirement could be met by different individual ship capabilities. For example, assume the total P_N requirement is 0.8. With a fixed number

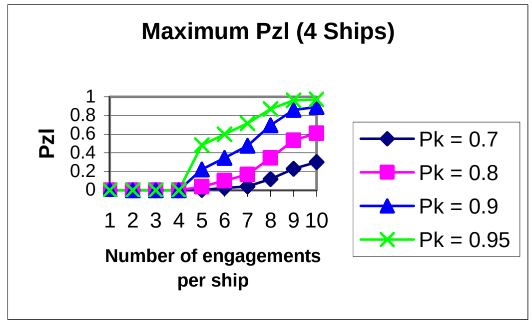

Figure 7.

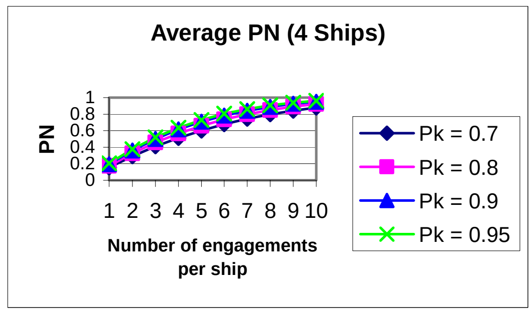

Figure 8.

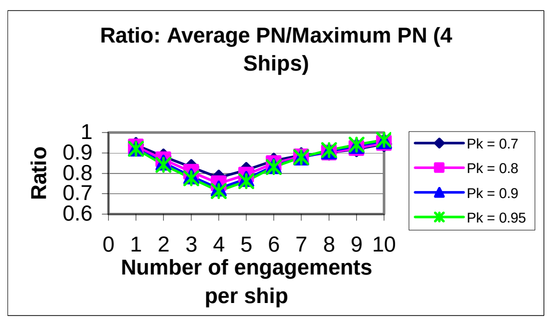

Figure 9.

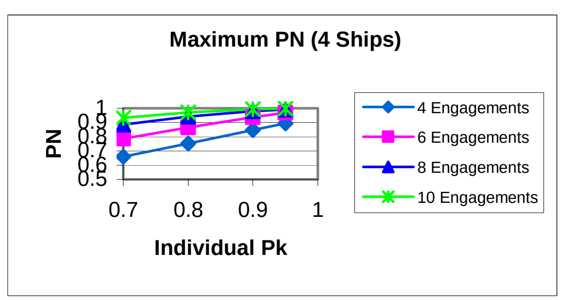

Figure 10.

of ships (four) engaging 17 targets using perfect coordination, the needed value for total P_N could be met in several ways. The four ships could engage: (1) four targets each and have an individual effectiveness of 0.9, or (2) six targets each with an individual engagement effectiveness of about 0.72, or (3) more targets engaged by each ship with even lower P_k values. Figure 11 shows the corresponding results for the uncoordinated case; similar options are also apparent here.

Figure 12 illustrates how the probability of no leakers depends on the individual engagement probabilities and the number of engagements each ship can prosecute (same 4 ships and 17 targets). As is apparent, the individual P_k value must be high to result in a non-negligible Pzl. The "4 Engagements" values are all zero since the ships could only engage, at most, 16 of the 17 targets in the raid.

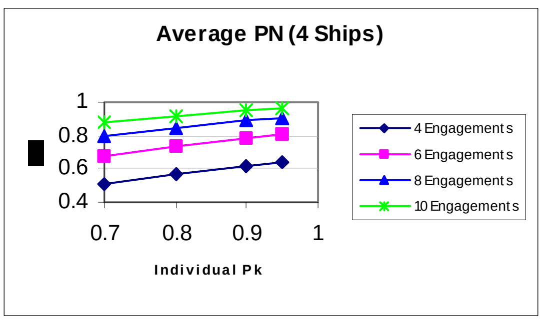

Figure 11.

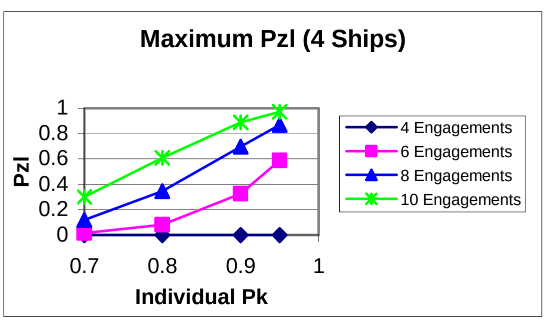

Figure 12.

Conclusion

This simple analysis demonstrates that the overall defensive effectiveness of a system of ships can potentially be generated in several ways. System coordination processes must be examined in the overall context of actual requirements and individual ship capabilities. As was shown, even perfect coordination may not significantly increase the expected overall effectiveness of a system of systems. The contribution of coordination is also difficult to quantify accurately, as is its cost.

The requirements of the individual ship systems must also be viewed from a total system vantage point. The development of a comprehensive and precise battle management and coordination system relying on effective and full communications among platforms may not be the most cost efficient method of generating total system effectiveness. Less complicated coordination schemes using additional or more capable units could possibly provide a more flexible and robust option. The overall value and required sophistication of battle management for TBMD (or other process with multi-system engagements of multiple targets) are still to be specifically determined. In addition, the determination and expression of needed capabilities (such as in system requirements documents) should be made only in a total system context.